

Chapter – 4 : Transformation

Matrices and matrix operations are very important mathematical tools which are used in computer graphics very extensively. Compact notation for describing sets of data and sets of equations.

4.1 Introduction to Matrix and It's Operation

A **matrix** is a rectangular table or array of abstract quantities which may be numbers arranged in rows and columns. It is denoted by the alphabets in block letter. The array of numbers below is an example of a matrix.

$$\mathbf{A} = \begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix}$$

The number of rows and columns that a matrix has is called its **dimension** or its **order**. By convention, rows are listed first; and columns, second. Thus, we would say that the dimension (or order) of the above matrix is 3 x 4, meaning that it has 3 rows and 4 columns.

Numbers that appear in the rows and columns of a matrix are called **elements** or **entries** of the matrix. In the above matrix, the element in the first column of the first row is 21; the element in the second column of the first row is 62; and so on.

Statisticians use symbols to identify matrix elements and matrices.

Matrix elements: Consider the matrix below, in which matrix elements are represented entirely by symbols.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}$$

By convention, first subscript refers to the row number; and the second subscript, to the column number. Thus, the first element in the first row is represented by A_{11} . The second element in the first row is represented by A_{12} . And so on, until we reach the fourth element in the second row, which is represented by A_{24} .

Matrices: There are several ways to represent a matrix symbolically. The simplest is to use a boldface letter, such as **A**, **B**, or **C**. Thus, **A** might represent a 2 x 4 matrix, as illustrated below.

$$\mathbf{A} = \begin{bmatrix} 11 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \end{bmatrix}$$

Another approach for representing matrix **A** is: $\mathbf{A} = [a_{ij}]$ where $i = 1, 2$ and $j = 1, 2, 3, 4$

This notation indicates that **A** is a matrix with 2 rows and 4 columns. The actual elements of the array are not displayed; they are represented by the symbol a_{ij} .

Many times one of the indices i or j is 1. Then matrix with dimension $m \times 1$ is termed as column matrix while with dimension $1 \times n$ is termed as row matrix.

4.1.1 Various types of Matrix

Square Matrix: A matrix is said to be a square matrix if the no. of rows and columns in the matrix are same. That is $m = n$.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 5 & 6 & 7 \\ 6 & 3 & 2 \\ 7 & 2 & 1 \end{bmatrix}$$

Unit Matrix: A matrix in which leading diagonal elements are 1 and all other elements are 0. That is $a_{ij} = 1$ (for $i = j$) and $a_{ij} = 0$ (for $i \neq j$). It is always denoted by $\mathbf{I}$.

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If we multiply the unit matrix with any other matrix the resultant matrix remain the same matrix as it is. That is $\mathbf{A} \cdot \mathbf{I} = \mathbf{A}$. (both matrix should be of equal dimensions) Then the product of $\mathbf{A}$ and $\mathbf{I}$ will be $\mathbf{A}$ itself.

$$\mathbf{A} = \begin{bmatrix} 5 & 6 & 7 \\ 6 & 3 & 2 \\ 7 & 2 & 1 \end{bmatrix} \quad \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Symmetric Matrix: A square matrix in which all $a_{ij} = a_{ji}$ is called a symmetric matrix. Following is an example of such matrix.

$$\mathbf{A} = \begin{bmatrix} 5 & 6 & 7 \\ 6 & 3 & 2 \\ 7 & 2 & 1 \end{bmatrix}$$

Skew Symmetric Matrix: A square matrix in which all $a_{ij} = -a_{ji}$ is called a skew symmetric matrix. Following is an example of such matrix.

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 7 \\ -2 & 7 & 2 \\ -7 & -2 & 1 \end{bmatrix}$$

Transposition Matrix: A matrix in which all rows are converted to columns and all columns are converted to rows is known as a transposition matrix of the original matrix. Which means a transposition matrix of $(m \times n)$ direction will be of $(n \times m)$. it is denoted by a superscript T .

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Orthogonal Matrix: Matrix $\mathbf{A}$ is said to be an orthogonal matrix if it produces unit matrix when it is multiplied by its own transpose.

$$\mathbf{A} \cdot \mathbf{A}^T = \mathbf{I}$$

Zero / Null Matrix: If a matrix $\mathbf{A}$ is having all elements as 0 then the matrix $\mathbf{A}$ is said to be a Zero or Null Matrix.

Diagonal Matrix: A diagonal matrix is a special kind of symmetric matrix. It is a symmetric matrix with zeros in the off-diagonal elements. Two diagonal matrices are shown below.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that the diagonal of a matrix refers to the elements that run from the upper left corner to the lower right corner.

Scalar Matrix: A scalar matrix is a special kind of diagonal matrix. It is a diagonal matrix with equal-valued elements along the diagonal. Two examples of a scalar matrix appear below.

$$\mathbf{A} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

4.2 Matrix Operations

The basic operations performed on the Matrices are as below:

4.2.1 Scalar Multiplication

It is a unary operation. Which mean it can be performed only on one matrix at a time. If matrix A is of m X n dimensions and c is a constant number or a scalar than scalar multiplication of the matrix A with scalar c would be obtained by multiplying all the entries of matrix with the scalar c, which means for all a_{ij} with c where $1 \leq i \leq m$ and $1 \leq j \leq n$. That is $c \cdot A = ca_{ij}$.

For e.g.

$$C = 5 \text{ and } A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 5 & 3 \\ 9 & 4 & 5 \end{bmatrix} \quad \text{Then } c \cdot A = \begin{bmatrix} 10 & 15 & 25 \\ 35 & 25 & 15 \\ 45 & 20 & 25 \end{bmatrix}$$

4.2.2 Matrix Addition and Subtraction

For performing addition and subtraction of two matrices, they must be of same dimension. If matrices A and B are of same dimensions, then the summation of these two matrices would be obtained by adding all corresponding entries of matrices together, which can be represented as follows:

$$A + B = a_{ij} + b_{ij} \text{ where } 1 \leq i \leq m \text{ and } 1 \leq j \leq n.$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 & 3 \\ 3 & 4 & 1 \\ 5 & 1 & 6 \end{bmatrix} \quad A + B = \begin{bmatrix} 6 & 4 & 6 \\ 7 & 9 & 7 \\ 12 & 9 & 8 \end{bmatrix}$$

Similarly the subtraction $A - B$ can be obtained by subtracting all entries of matrix B from the corresponding entry of matrix A.

$$A - B = a_{ij} - b_{ij} \text{ where } 1 \leq i \leq m \text{ and } 1 \leq j \leq n.$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 & 3 \\ 3 & 4 & 1 \\ 5 & 1 & 6 \end{bmatrix} \quad A - B = \begin{bmatrix} -4 & 0 & 0 \\ 1 & 1 & 5 \\ 2 & 7 & -4 \end{bmatrix}$$

4.2.3 Matrix Multiplication

Two matrices can be multiplied to each other on one condition that is no. of columns of the matrix on the left side should be equal to no. of rows of matrix on the right side of the multiplication operator. So the matrix A of m X n dimension can be multiplied to matrix B of p X q dimensions only if n = p. the product of two matrices is denoted by (·) dot operator. So it can be written as follows:

$$\mathbf{A}_{m \times n} \times \mathbf{B}_{p \times q} = \mathbf{C}_{m \times q} \text{ (only if } n = p \text{)}$$

It can be derived by,

$$ab_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{pj}$$

For e.g.

$$\begin{aligned} \mathbf{A}_{2 \times 3} \times \mathbf{B}_{3 \times 2} &= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ -2 & -1 \\ 0 & 4 \end{bmatrix} \\ \mathbf{A} \times \mathbf{B}_{2 \times 2} &= \begin{bmatrix} 1 \times 0 + 0 \times -2 + -1 \times 0 & 1 \times 3 + 0 \times -1 + -2 \times 4 \\ 0 \times 0 + 3 \times -2 + -1 \times 0 & 0 \times 3 + 3 \times -1 + -1 \times 4 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -5 \\ -6 & -7 \end{bmatrix} \end{aligned}$$

4.2.4 Determinant of Matrix

The determinant of a square matrix is a function depending upon the scalar of matrix. It is denoted by $\det(A)$ or $|A|$.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Then the determinant is denoted by $|A|$ and given by

$$|A| = (a_{11} \cdot a_{22} - a_{12} \cdot a_{21})$$

Where as if the matrix is of 3X3 dimensions then the determinant can be given as:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A| = a_{11} (a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) - a_{12} (a_{21} \cdot a_{33} - a_{23} \cdot a_{31}) + a_{13} (a_{21} \cdot a_{32} - a_{22} \cdot a_{31})$$

The sign pattern to be consider is as follow for the calculation of the determinants of different entries.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

If the determinant of the matrix is a non-zero then it is called non-singular matrix, and if it

is zero then it is called singular matrix.

If A is a square matrix and let its cofactor matrix be $[A_{ij}]$, then the transpose of $[A_{ij}]$ that is $[A_{ij}]^T$ is called the adjoint of matrix A and it is denoted by adj.A .

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{Then the adj.A will be} \quad \text{adj.A} = \begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}$$

4.2.5 Inverse of Matrix

Let A and B two matrices of order $m \times m$, then they are called inverse of each other if,

$$A \cdot B = B \cdot A = I$$

$$\text{An inverse of matrix A is denoted by } A^{-1} = \frac{1}{|A|} \cdot \text{adj.A}$$

An inverse of matrix is unique, since an inverse of matrix exists only if non-singular; it is required to check it's singularity before finding the inverse.

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 5 & 3 \\ 9 & 4 & 5 \end{bmatrix} \quad \text{it is}$$

Then the $|A| = 2(25-12) - 3(35-27) + 5(28-45) = 26 - 24 + (-17) = -15$; which is indicating that the matrix A is non-singular matrix, so it is invertible.

$$\text{adj.A} = \begin{bmatrix} \begin{vmatrix} 5 & 3 \\ 4 & 5 \end{vmatrix} & -\begin{vmatrix} 3 & 5 \\ 4 & 5 \end{vmatrix} & \begin{vmatrix} 3 & 5 \\ 5 & 3 \end{vmatrix} \\ -\begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 9 & 5 \end{vmatrix} & -\begin{vmatrix} 7 & 5 \\ 7 & 3 \end{vmatrix} \\ \begin{vmatrix} 7 & 5 \\ 9 & 4 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 9 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 7 & 5 \end{vmatrix} \end{bmatrix} \quad \text{adj.A} = \begin{bmatrix} 13 & 5 & -16 \\ -8 & -35 & 14 \\ 17 & 19 & -11 \end{bmatrix}$$

So the inverse of matrix A can be given as A^{-1} is given as

$$A^{-1} = \begin{bmatrix} -13/15 & -1/3 & 16/15 \\ +8/15 & 7/3 & -14/15 \\ -17/15 & -19/15 & 11/15 \end{bmatrix}$$

4.2.5 Method of Reduction (Gauss Elimination Method)

The method of find the inverse is too lengthy to perform. The method of reduction is known as Gauss Elimination Method. It works on the principle "If a non-singular matrix A is converted into an identity matrix by a series of row and/or column operations, then the same series of operations performed on the identity matrix converts it into the inverse matrix of A."

Let A be the m X m non-singular matrix to be inverted. The augmented matrix would be,

$$\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{1m} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{2m} & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{mm} & 0 & 0 & 1 \end{array} \right]$$

Let A be the m X m non-singular matrix to be inverted. The augmented matrix would be,

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & a_{11} & a_{12} & a_{1m} \\ 0 & 1 & 0 & a_{21} & a_{22} & a_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & a_{m1} & a_{m2} & a_{mm} \end{array} \right]$$

The matrix $B = A^{-1}$.

The row and column operations will be performed in such a way that the element of first column and first row should be converted to 1 and all other element of the same row column will become 0. This process should be repeated to constitute the unit matrix. Check out the following example.

Find A^{-1} for the matrix A by using Gauss Elimination Method.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 5 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

Solution: First of all we have to find the determinant of matrix A, for checking whether it is a non-singular matrix or not.

$$|A| = 1(5-18) - 2(2-3) + 5(12-5) = -13 + 2 + 35 = 24 \neq 0, \text{ matrix A is non-singular matrix.}$$

Now we have to perform row and column operation on the matrix to find the inverse.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 6 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 1 & -7 & -2 & 1 & 0 \\ 0 & 4 & -4 & -1 & 0 & 1 \end{array} \right] \quad \mathbf{R_2 - 2R_1, R_3 - R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 1 & -7 & -2 & 1 & 0 \\ 0 & 0 & 24 & 7 & -4 & 1 \end{array} \right] \quad \mathbf{R_3 - 4R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 1 & -7 & -2 & 1 & 0 \\ 0 & 0 & 1 & 7/24 & -1/6 & 1/24 \end{array} \right] \quad \mathbf{(1/24)R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -11/24 & -5/6 & -5/24 \\ 0 & 1 & 0 & 1/24 & -1/6 & 7/24 \\ 0 & 0 & 1 & 7/24 & -1/6 & 1/24 \end{array} \right] \quad \mathbf{R_2 + 7R_3, R_1 - 5R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -25/24 & -1/2 & -7/24 \\ 0 & 1 & 0 & 1/24 & -1/6 & 7/24 \\ 0 & 0 & 1 & 7/24 & -1/6 & 1/24 \end{array} \right] \quad \mathbf{R_1 - 2R_2}$$

So the inverse of the matrix A would be,
$$A^{-1} = \begin{bmatrix} -25/24 & -1/2 & -7/24 \\ 1/24 & -1/6 & 7/24 \\ 7/24 & -1/6 & 1/24 \end{bmatrix}$$

4.3 Transformation

In 2D graphics we are using different objects like lines, curves, polygons, circle, etc.; all these objects need to be altered at many situations. The alterations which can be applied are moving them, rotating them as well as enlarging or reducing them. These processes are termed as Scaling, Rotations and Translations in computer Graphics. For performing these actions it needs certain basic geometry, so they are also known as geometric transformations.

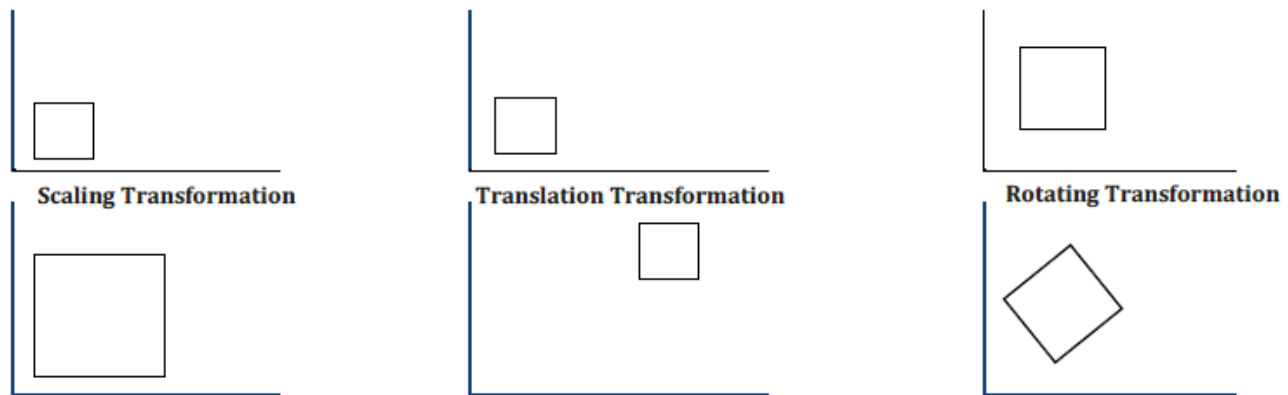


Fig. 4.1 Various Geometric Transformations in 2D

4.3.1 Scaling

Scaling transformation is used to alter the size of the given object, which is defined in term of the scaling factor. The scale factor defines that how much bigger or smaller the object would look after the scaling. To alter the object in horizontal direction the scale factor S_x is used whereas to alter the object in the vertical direction the scale factor S_y is used. The scaling transformation is performed by multiplying the coordinates (x,y) with the scaling factor S_x and S_y respectively. Mathematical representation of transformation of the original coordinates (x,y) into scaled coordinates (x',y') is:

$$x' = S_x \cdot x \text{ and } y' = S_y \cdot y$$

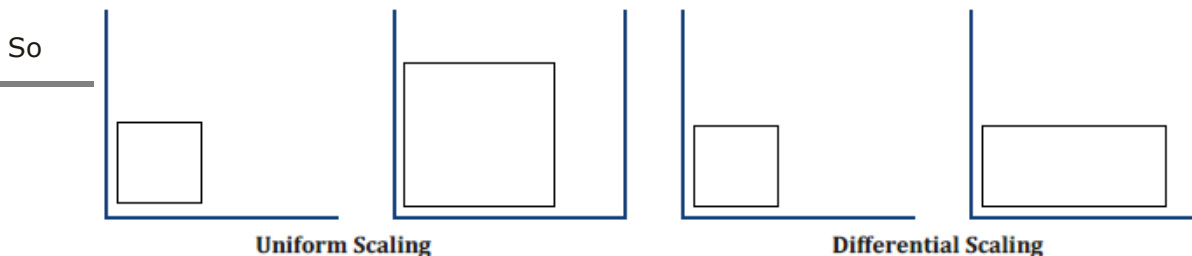
To scale a polygon object, the transformation is performed on all the vertices of the polygon. To enlarge the size of the polygon horizontally, the X-coordinates of all the vertices need to be multiplied with the enlarging factor. Similarly for reducing the size in horizontal direction all the vertices should be multiplied with the reduction factor (i.e. for reduce the size to half we have to multiply the coordinates with 0.5). It is similar with the vertical alterations whether enlarging or reducing. So it can be given in form of matrix as below:

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

Which can be written as $P' = P \cdot S$, where P' is matrix of 1X2 dimensions of the scaled coordinates while P is the matrix of 1X2 dimensions of the original coordinates and S is the matrix of 2X2 dimensions of the scaling transformations. Here the scaling coordinates will always be

positive.
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scaling factors S_x , S_y are greater than 1 then the object will be enlarged in horizontal or vertical directions respectively. If these values are between 0 and 1 then the object will be reduced in respective directions. And if the values are 1 then the scaling matrix will be a unique matrix as well as there will be no change in the object. When the scaling is equal in both horizontal and vertical directions then the value of $S_x = S_y$. This type of scaling transformation is known as uniform scaling whereas in other situations it is known as differential scaling.

When we are performing the scaling transformation on the object, we have to check that after performing the transformation the object does not lose its position. In all the graphics applications it requires to keep the object at its place which may be represented as (x_k, y_k) will not be changed after the transformation. So this transformation can be given by the equations:

$$\mathbf{x}' = \mathbf{x}S_x + (\mathbf{x}_k - \mathbf{x}_kS_x) \text{ and } \mathbf{y}' = \mathbf{y}S_y + (\mathbf{y}_k - \mathbf{y}_kS_y) \text{ or}$$

$$\mathbf{x}' = \mathbf{x}S_x + \mathbf{x}_k(1 - S_x) \text{ and } \mathbf{y}' = \mathbf{y}S_y + \mathbf{y}_k(1 - S_y)$$

It can be written in matrix form as below:

$$[\mathbf{x}' \ \mathbf{y}'] = [\mathbf{x} \ \mathbf{y}] \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} + [\mathbf{x}_k(1 - S_x) \ \mathbf{y}_k(1 - S_y)]$$

Where $\mathbf{x}_k(1 - S_x)$ and $\mathbf{y}_k(1 - S_y)$ are constant terms which are added to the original scale transformations.

Above equation can work well with the objects with straight lines whereas if the object is having curves like circle and ellipse this equation can't be used. In such situations parameter like radius for the circle and semi-major and semi-minors in case of ellipse will be used.

4.3.2 Translation

Translation of an object means moving the object from one place to other in 3 different ways as below:

- Translation in horizontal direction- parallel to horizontal axis – X
- Translation in vertical direction- parallel to vertical axis – Y
- Translation in both the direction simultaneously.

For understanding the translation let us assume we have a point (x, y) to be translated in horizontal direction, means it will be parallel to the X-axis by T_x units. So this point can be obtained by adding the T_x to X-coordinate, so the coordinates after transformation will be,

$$\mathbf{x}' = \mathbf{x} + T_x \text{ and } \mathbf{y}' = \mathbf{y}$$

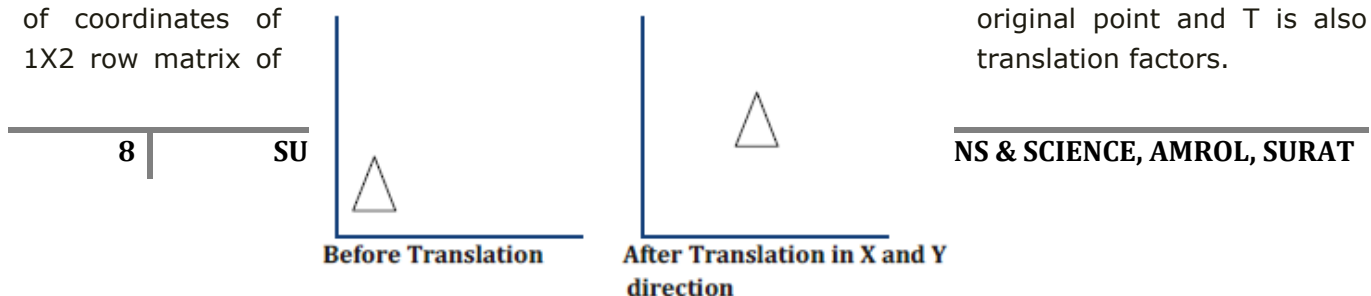
Similarly, if we translate point (x, y) in vertical direction parallel to the Y-axis by T_y unit, so the coordinates after translation can be given as:

$$\mathbf{x}' = \mathbf{x} \text{ and } \mathbf{y}' = \mathbf{y} + T_y$$

In both above mentioned equations T_x and T_y are translation factors for translating a point in the X and Y directions respectively. In general it can be written as:

$$(\mathbf{x}', \mathbf{y}') = (\mathbf{x} + T_x, \mathbf{y} + T_y) \text{ or } [\mathbf{x}' \ \mathbf{y}'] = [\mathbf{x} \ \mathbf{y}] + [T_x \ T_y] \quad \text{or} \quad \mathbf{P}' = \mathbf{P} + \mathbf{T}$$

Where $\mathbf{P}'$ is a 1X2 row matrix of coordinates of the translated point, $\mathbf{P}$ is the 1X2 row matrix of coordinates of original point and $\mathbf{T}$ is also translation factors.



The translation transformation is called a rigid body transformation since the Euclidian distance between any two coordinates are never changes during the translation.

Here the translation is performed on each and every vertices of the object to translate the object. When new vertices are found the object is redrawn with the translated points. Other objects like circle, ellipse etc. when there is no straight lines in the objects certain parameters are used for performing the translations like centre point of circle.

4.3.3 Rotation about Origin

Rotation is the mostly used transformation. In this transformation an object moves on the circular path about its origin. It requires two major parameters direction of rotation and angle of the rotation. The rotation can be performed in anti-clockwise or clockwise directions by a given angle Θ .

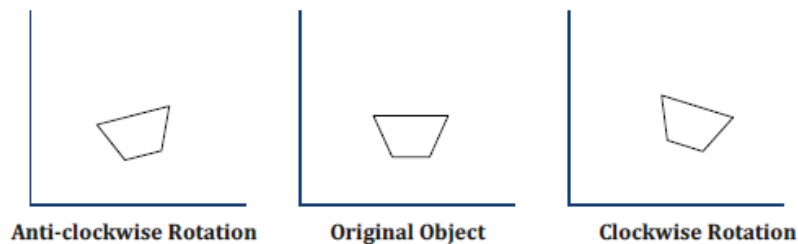


Fig. 4.4 Rotation Transformation

The rotation in 2D graphics is performed around some origin point which is known as rotation point. This rotation can also be described as rotation around an axis which is called rotation axis. This rotation axis passes through rotation point and is perpendicular to the XY-plane.

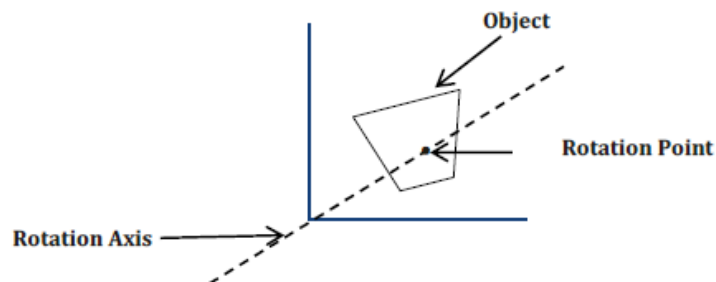


Fig. 4.5 Rotation about an arbitrary point or rotation about axis

Let's assume a point (x,y) is to be rotate anti-clockwise. In fig.4.6 the point (x',y') is rotated point, as a result of rotation transformation in anti-clockwise direction with angle Θ relative to the origin $(0,0)$. θ is an angle of point P from X-axis. So according to simple trigonometric principles; value of x and y can be obtained as:

$$x = r \cos \theta \quad y = r \sin \theta \quad \dots (1)$$

Where r is the length of OP , or the distance of point P from the origin $(0,0)$.

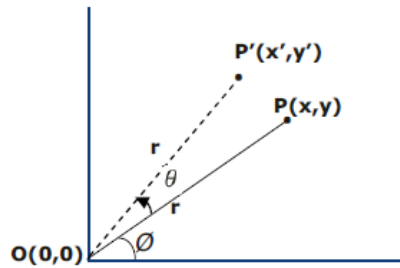


Fig. 4.5: Anti-clockwise Rotation of a point

$$x' = r \cos (\theta + \theta), \quad y' = r \sin (\theta + \theta) \quad \dots (2)$$

$$x' = r \cos \theta \cos \theta - r \sin \theta \sin \theta \quad \text{and} \quad y' = r \cos \theta \sin \theta + r \sin \theta \cos \theta \quad \dots (3)$$

When we implement value of equation (1) in (3), we get

$$x' = x \cos \theta - y \sin \theta \quad \text{and} \quad y' = x \sin \theta + y \cos \theta \quad \dots (4)$$

In the matrix notation it can be written as $\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad \dots (5)$

$$P' = P \cdot R_a \quad \dots (6)$$

In equation (6), R_a is the rotation transformation matrix for rotating a point P in anti-clockwise direction with angle θ relative to origin.

Similarly, we can derive the equation to rotate a point in clockwise direction by angle θ about the origin.

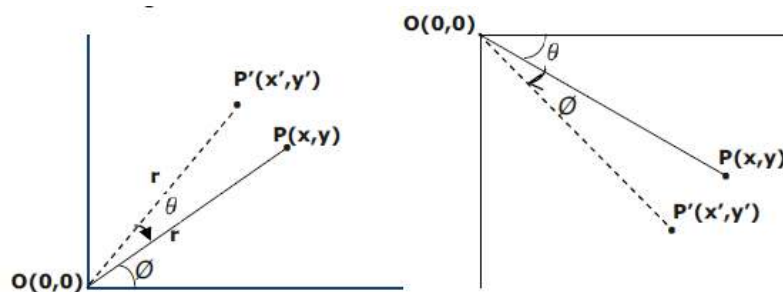


Fig. 4.6 : Clockwise Rotation of a point

Here we are rotating point $P(x, y)$ in the clockwise direction, and angle will be θ .

$$R_a = \begin{bmatrix} \cos -\theta & \sin -\theta \\ -\sin -\theta & \cos -\theta \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad \dots (7)$$

In equation (7), R_a is the rotation transformation matrix for rotating a point P in clockwise direction with angle θ relative to origin.

4.3.4 Shearing
